Name of College - S.S. College, Jehanabad
Dept - Mathematics

Topic - (Problem based on Plane)
Analytical geometry of 3-D

Class - B.Sc. (Sub + Hons)
Time - 1.00 P.M to 1.45 P.M
Date - 04-08-2020

By - Dr. Shree Nath Sharma

Problem :→ Find the Equation of the Plane passes through (2, -3, 1) and is perpendicular to the line joining the points (3, 4, -1) and (2, -1, 5)

Here Direction ratio of the Normal to the plane are $3-2,\ 4-(-1),\ \text{and}\ (-1-5)$

$\Rightarrow 1, 5, -6$

∴ Equation of plane be

$1 \times x + 5 \times y + (-6) z = d$

[Equation of plane $ax + by + cz = d$]

$x + 5y - 6z = d$

Since it passes through (2, -3, 1)

i.e $2 - 15 - 6 = d$

$-19 = d$

Required Equation $x + 5y - 6z = -19$

$\Rightarrow x + 5y - 6z + 19 = 0$ Hence the result

Problem :⇒ Show that the four points $(0,-1,-1)$, $(4,5,1)$, $(3,9,4)$ and $(-4,4,4)$ lie in a plane.

Solution :⇒ Since four points (x_1,y_1,z_1), (x_2,y_2,z_2), (x_3,y_3,z_3) and (x_4,y_4,z_4) are coplanar

if $$\begin{vmatrix} x_4-x_1 & y_4-y_1 & z_4-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0 \quad \text{(1)}$$

Here $x_1 = 0$, $y_1 = -1$, $z_1 = -1$

$x_2 = 4$, $y_2 = 5$, $z_2 = 1$

$x_3 = 3$, $y_3 = 9$, $z_3 = 4$

and $x_4 = -4$, $y_4 = 4$, $z_4 = 4$

Now $x_4 - x_1 = -4 - 0 = -4$

$y_4 - y_1 = 4 - (-1) = 5$

$z_4 - z_1 = 4 - (-1) = 5$

all these values

$x_2 - x_1 = 4 - 0 = 4$

$y_2 - y_1 = 5 - (-1) = 6$

$z_2 - z_1 = (1) - (-1) = 2$

$x_3 - x_1 = 3 - 0 = 3$

$y_3 - y_1 = 9 - (-1) = 10$

$z_3 - z_1 = 4 - (-1) = 5$

Putting in (1)

$$\begin{vmatrix} -4 & 4 & 3 \\ 5 & 6 & 10 \\ 5 & 2 & 5 \end{vmatrix}$$

$$= -4\begin{vmatrix} 6 & 10 \\ 2 & 5 \end{vmatrix} - 4\begin{vmatrix} 5 & 10 \\ 5 & 5 \end{vmatrix} + 3\begin{vmatrix} 5 & 6 \\ 5 & 2 \end{vmatrix}$$

$$= -4(30-20) - 4(25-50) + 3(10-30)$$

$$= -40 + 100 - 60$$

$$= 0$$

Thus the Given Four Points are coplanar

Problem : Find the Equation of plane through the points $P(1,1,1)$ $Q(3,-1,2)$ and $R(-3,5,-4)$.

Solution :→ Equation of plane through three points (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3) is Given by

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0 \quad \text{--- (1)}$$

Page

Here $x_1 = 1$, $y_1 = 1$, $z_1 = 1$

$x_2 = 3$, $y_2 = -1$, $z_2 = 2$

$x_3 = -3$, $y_3 = 5$, $z_3 = -4$

Now $x - x_1 = x - 1$, $y - y_1 = y - 1$, $z - z_1 = z - 1$

$x_2 - x = 3 - 1 = 2$, $y_2 - y_1 = -1 - 1 = -2$, $z_2 - z = 2 - 1 = 1$

$x_3 - x = -3 - 1 = -4$, $y_3 - y_1 = 5 - 1 = 4$, $z_3 - z = -4 - 1 = -5$

Putting all these values in (1)

$$\begin{vmatrix} x-1 & y-1 & z-1 \\ 2 & -2 & 1 \\ -4 & 4 & -5 \end{vmatrix} = 0$$

$$(x-1)\begin{vmatrix} -2 & 1 \\ 4 & -5 \end{vmatrix} - (y-1)\begin{vmatrix} 2 & 1 \\ -4 & -5 \end{vmatrix} + (z-1)\begin{vmatrix} 2 & -2 \\ -4 & 4 \end{vmatrix}$$

$\Rightarrow (x-1)(10-4) - (y-1)(-10+4) + (z-1)(8-8) = 0$

$\Rightarrow 6x - 6 + 6y - 6 + 12 - 4 = 0$

$6x + 6y = 12 \Rightarrow x + y - 2 = 0$

~~$\Rightarrow 6x + 6y + 4 = 0$~~